Chapter 26: Magnetism, Force and Field
Thursday October 13th

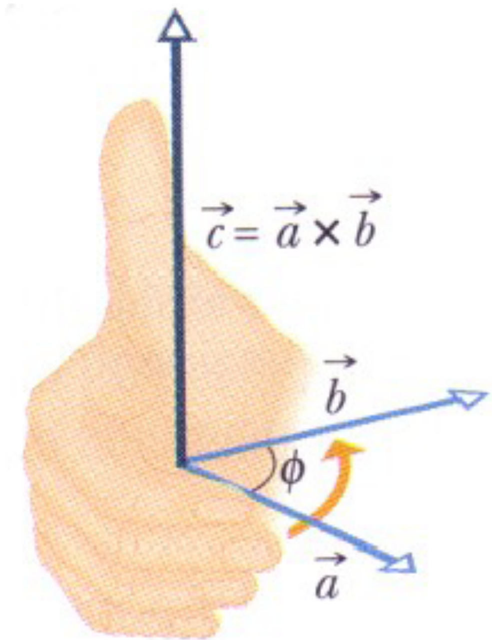
- **Cumulative mid-term exam next Thursday**
  (In-class – 75 minute, written exam)
- **No labs next week (resume following week)**
- **LONCAPA Monday, review Wednesday**

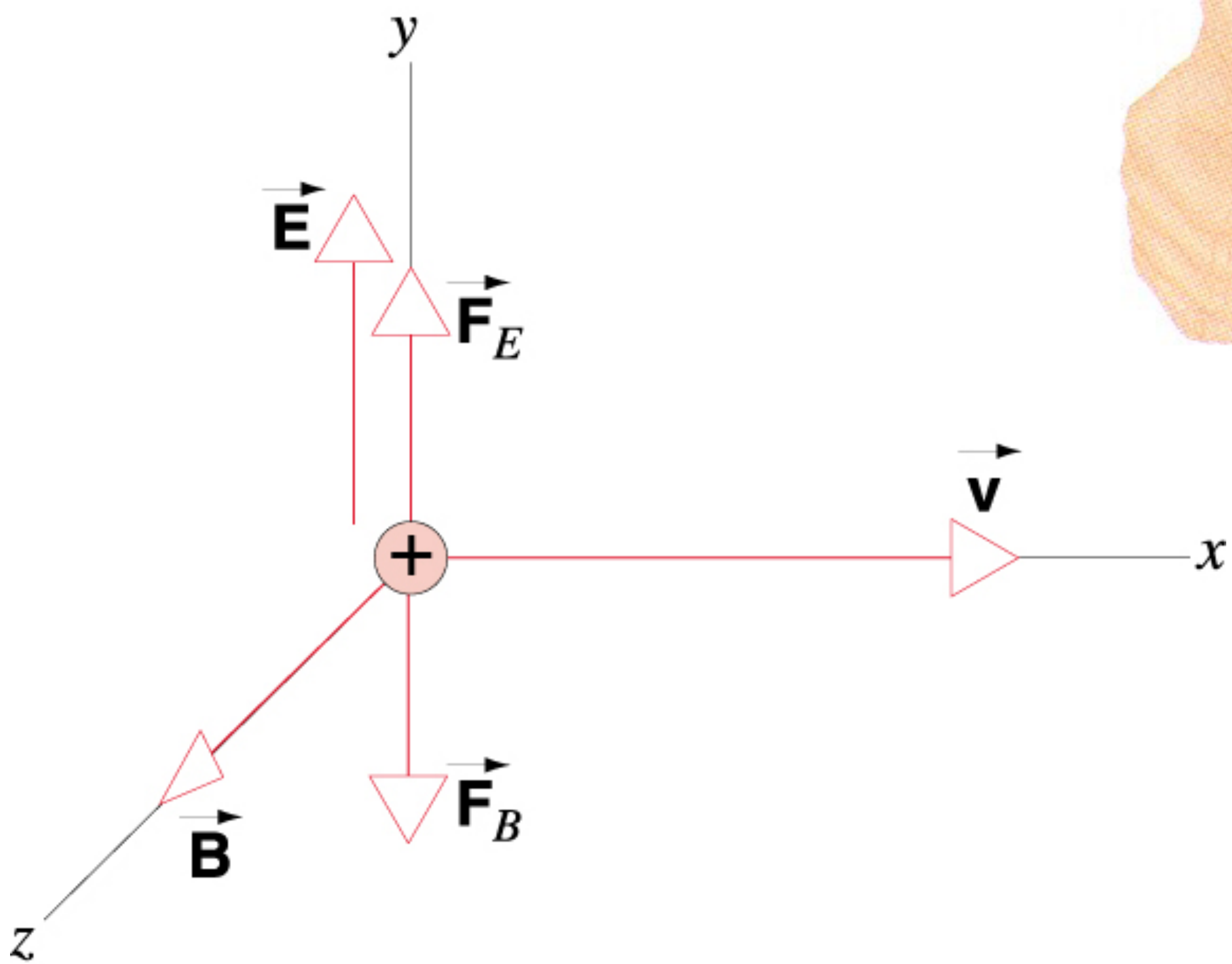
- Review of Lorentz force and force on a current carrying wire
- Sources of magnetic field
  - Moving charges
  - The field due to a current – the Biot-Savart law
- Examples

Reading: up to page 451 in the text book (Ch. 26)

The Lorentz Force

$$\vec{\mathbf{F}} = q\left[\vec{\mathbf{E}} + \left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)\right]$$





The Lorentz Force

•The velocity filter:

(undeflected trajectories in crossed E and B fields)

$$v = \frac{E}{B}$$

•Cyclotron motion:

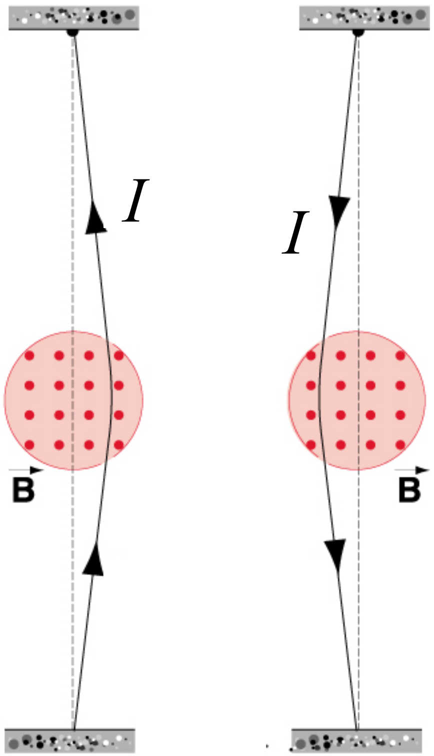
$$F_B = ma_r \qquad \Rightarrow \qquad |q|vB = m\frac{v^2}{r}$$

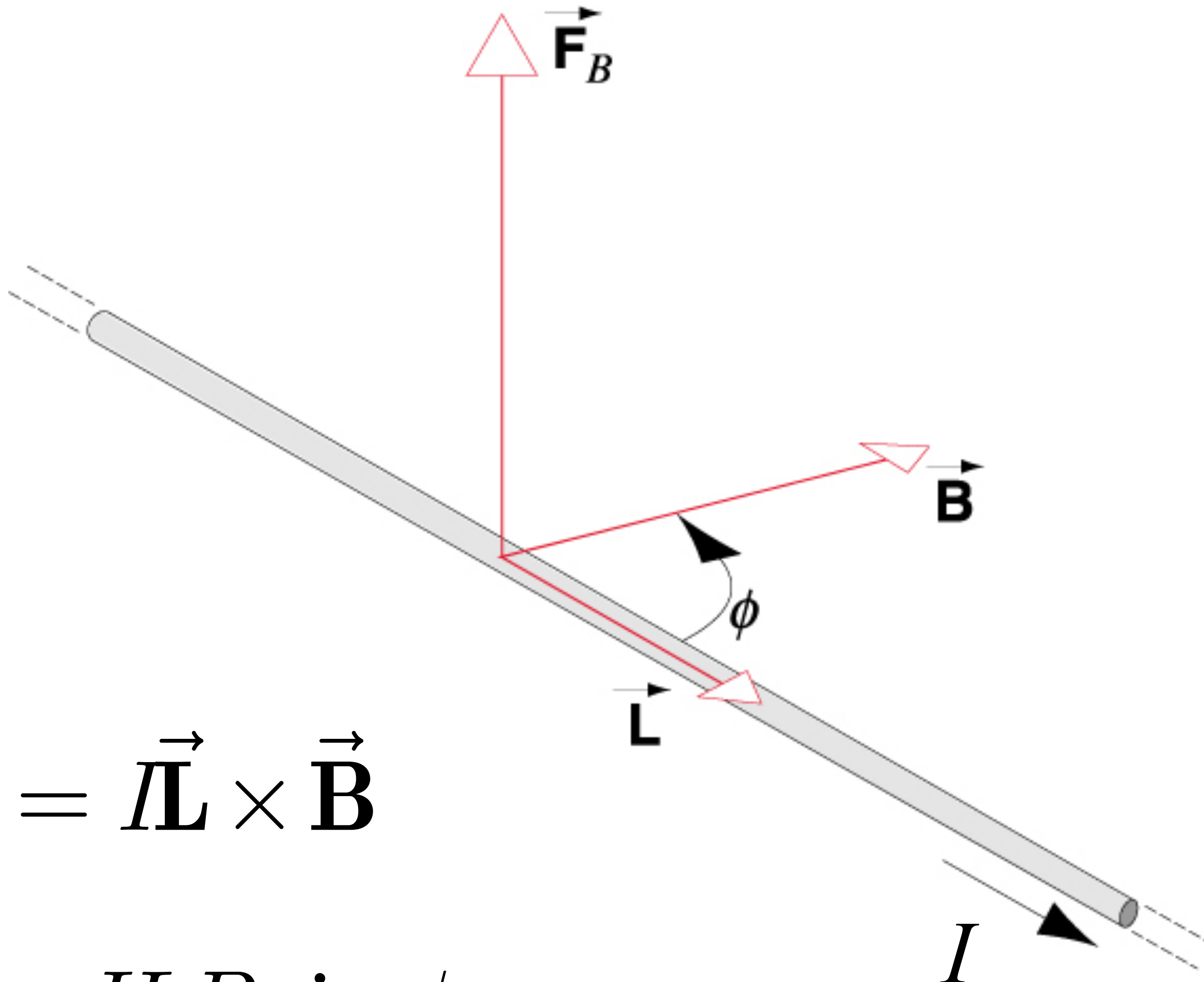
•Orbit radius: $$r = \frac{mv}{|q|B} = \frac{p}{|q|B}$$ momentum (p) filter

•Orbit frequency: $$\omega = 2\pi f = \frac{|q|B}{m}$$ mass detection

•Orbit energy: $$K = \tfrac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

The Magnetic Force on a Current-Carrying Wire

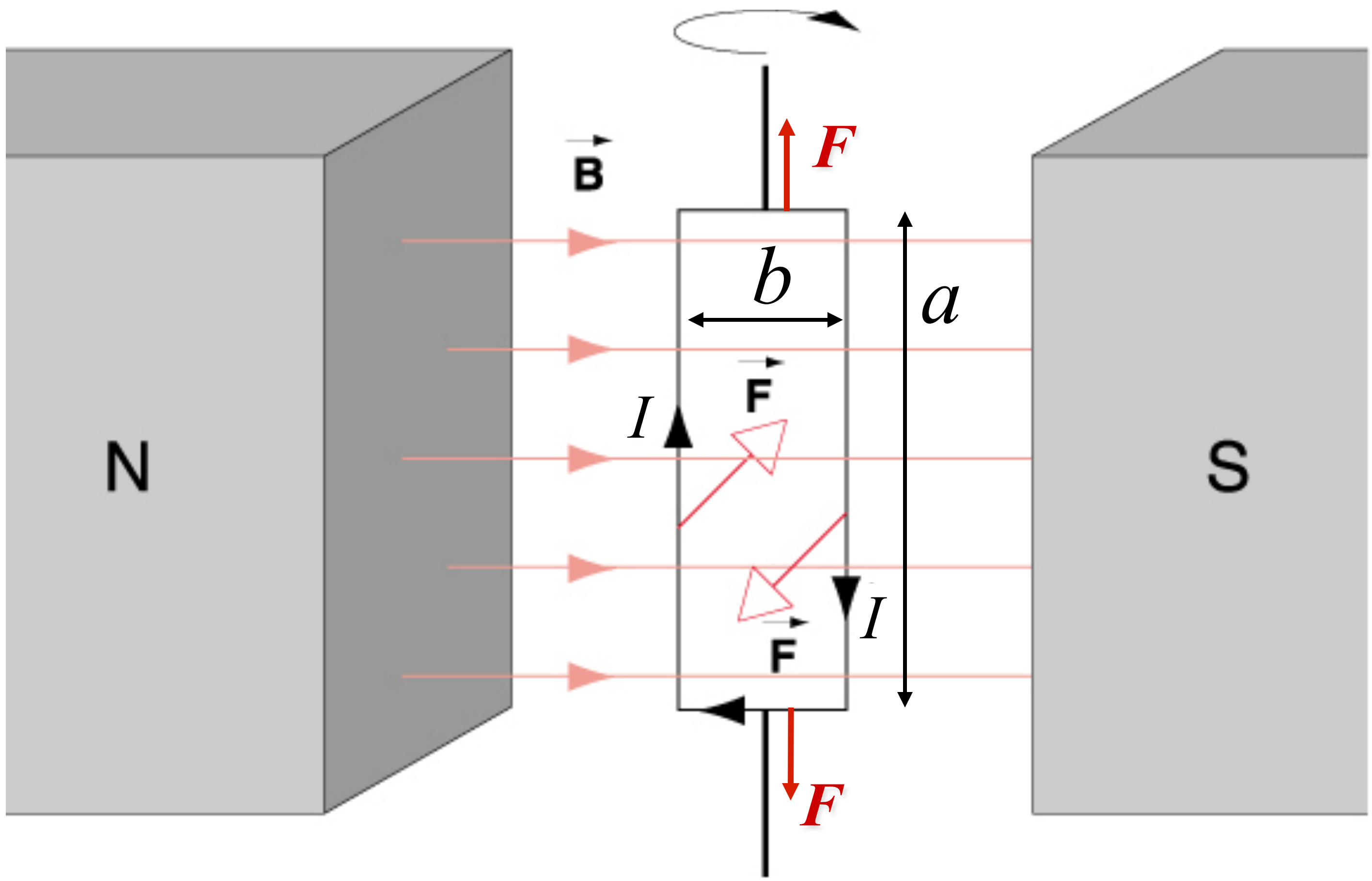




$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

$$= ILB \sin\phi$$

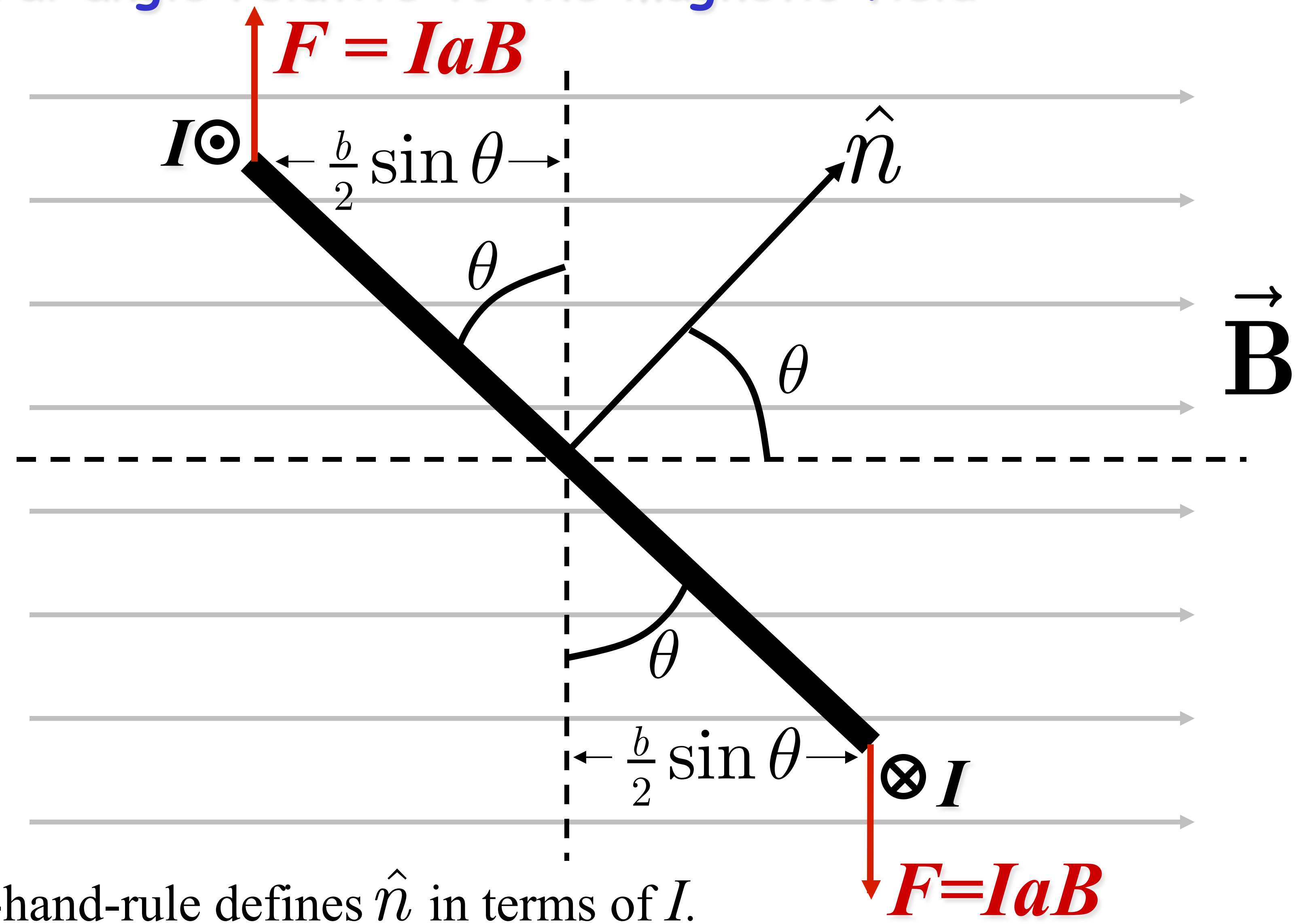
The Torque on a Current Loop



- Forces at the ends will always be in the plane of the loop.
- Consequently they produce no torque, and no net force.
- Force on sides do produce torque, but no net force.

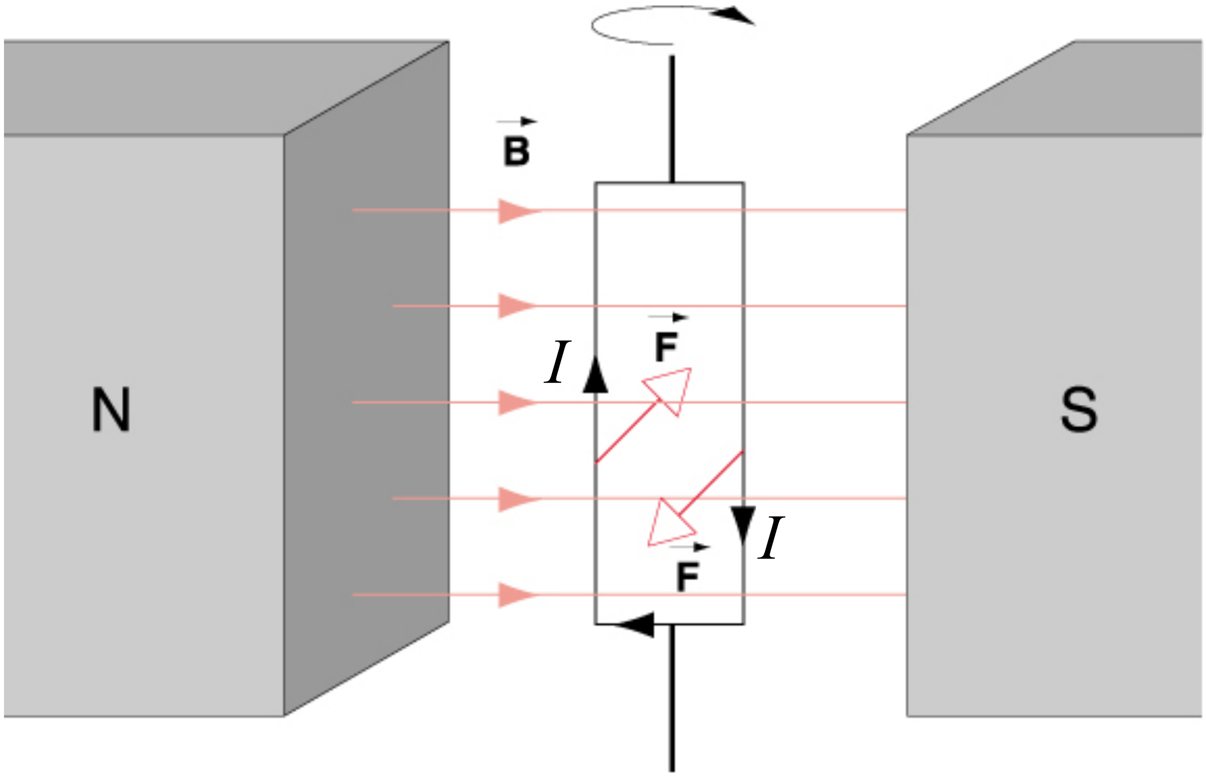
The Torque on a Current Loop

A view from the top, with the plane of the loop at some general angle relative to the magnetic field



Right-hand-rule defines $\hat{n}$ in terms of I.

The Torque on a Current Loop



$$\vec{\tau} = IA\left(\hat{n} \times \vec{B}\right)$$

$$= I\left(\vec{\mathbf{A}} \times \vec{B}\right)$$

$$= \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

Magnetic dipole moment: $\vec{\mu} = I\vec{\mathbf{A}}$

If the loop has N turns: $\vec{\mu} = NI\vec{\mathbf{A}}$

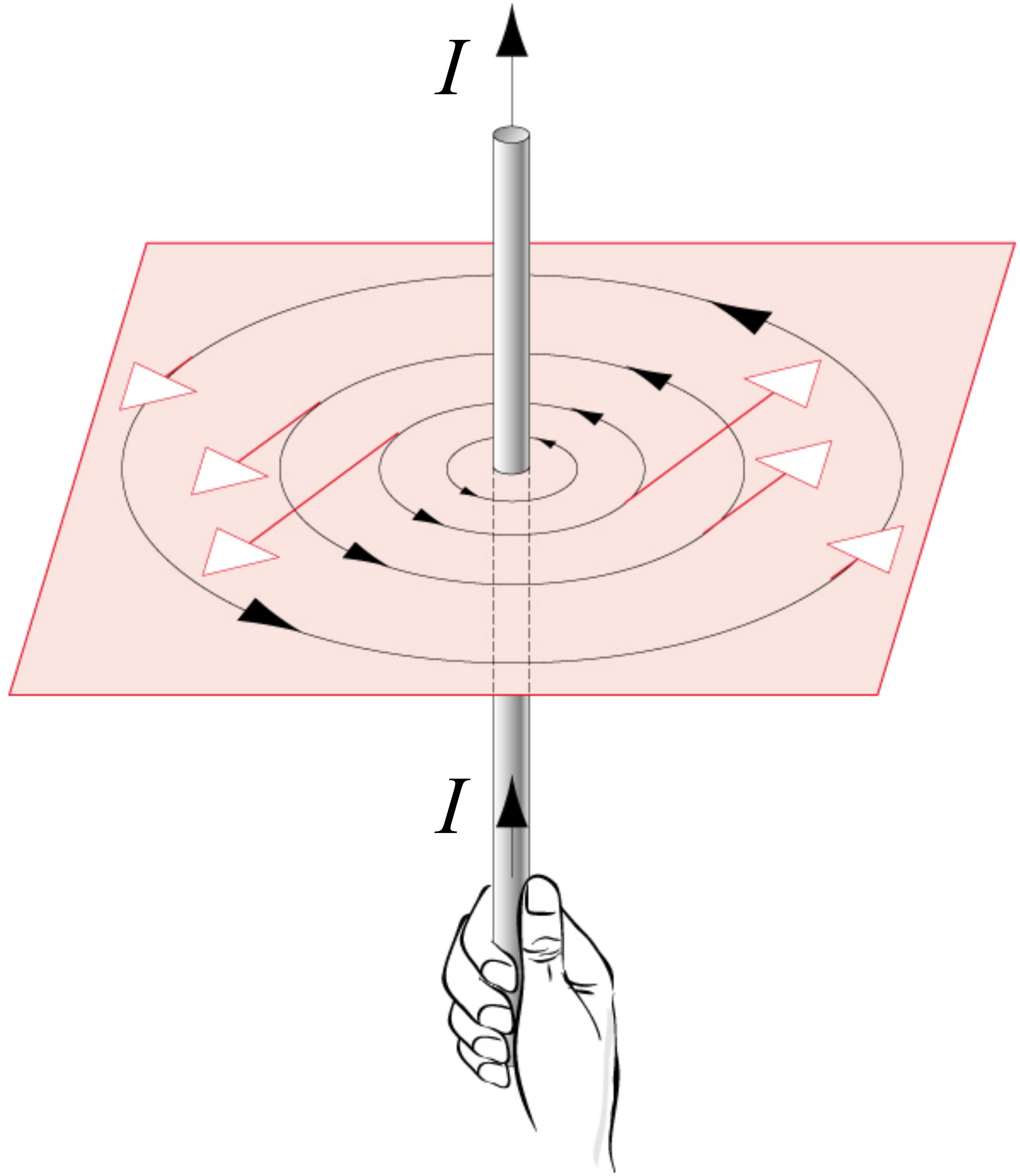
The magnetic field due to a wire in 3D

$$B = \frac{\mu_o I}{2\pi r}$$

Like Coulomb' s Law:
(For an infinite line charge)

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

$$\frac{1}{\varepsilon_o} \rightarrow \mu_o \qquad \lambda \rightarrow I$$

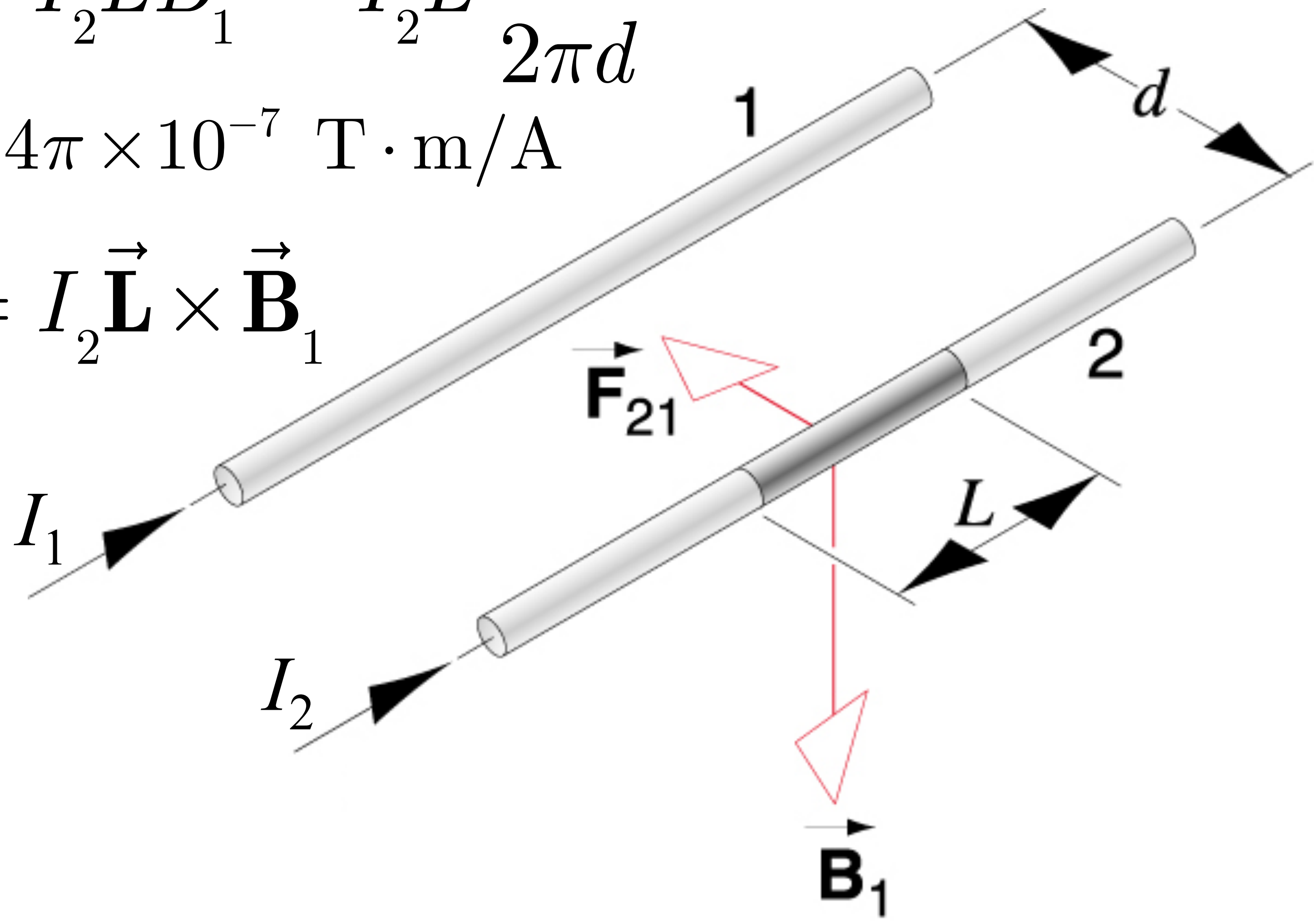


Ampere: magnetic force between two wires

$$F_{21} = I_2 L B_1 = I_2 L \frac{\mu_o I_1}{2\pi d}$$

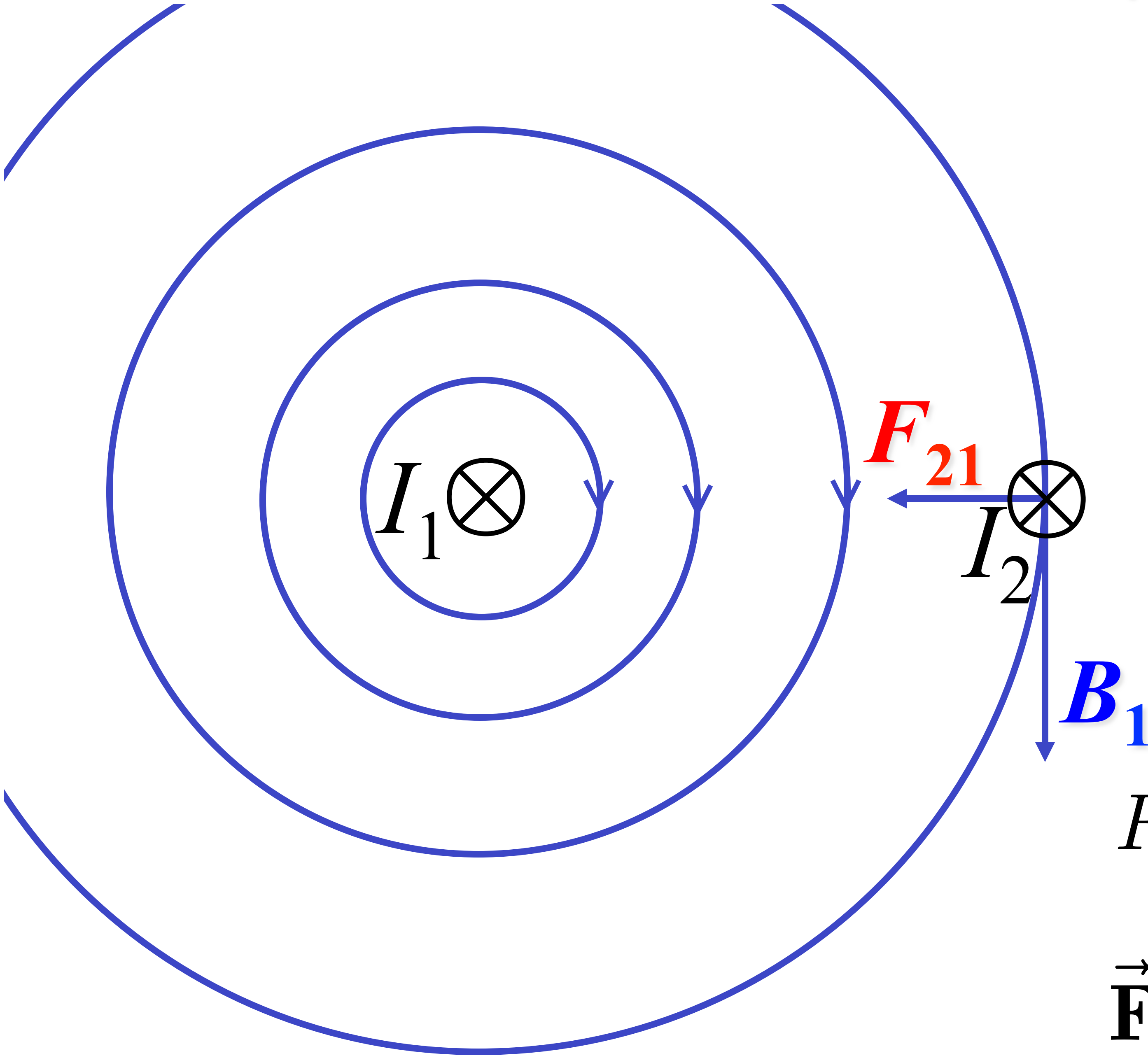
$$\mu_o = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

$$\vec{\mathbf{F}}_{21} = I_2 \vec{\mathbf{L}} \times \vec{\mathbf{B}}_1$$



μ_o chosen so that when $I_1 = I_2 = 1$ A, and $L = d = 1$ m, $F_{21} = 2\times10^{-7}$ N

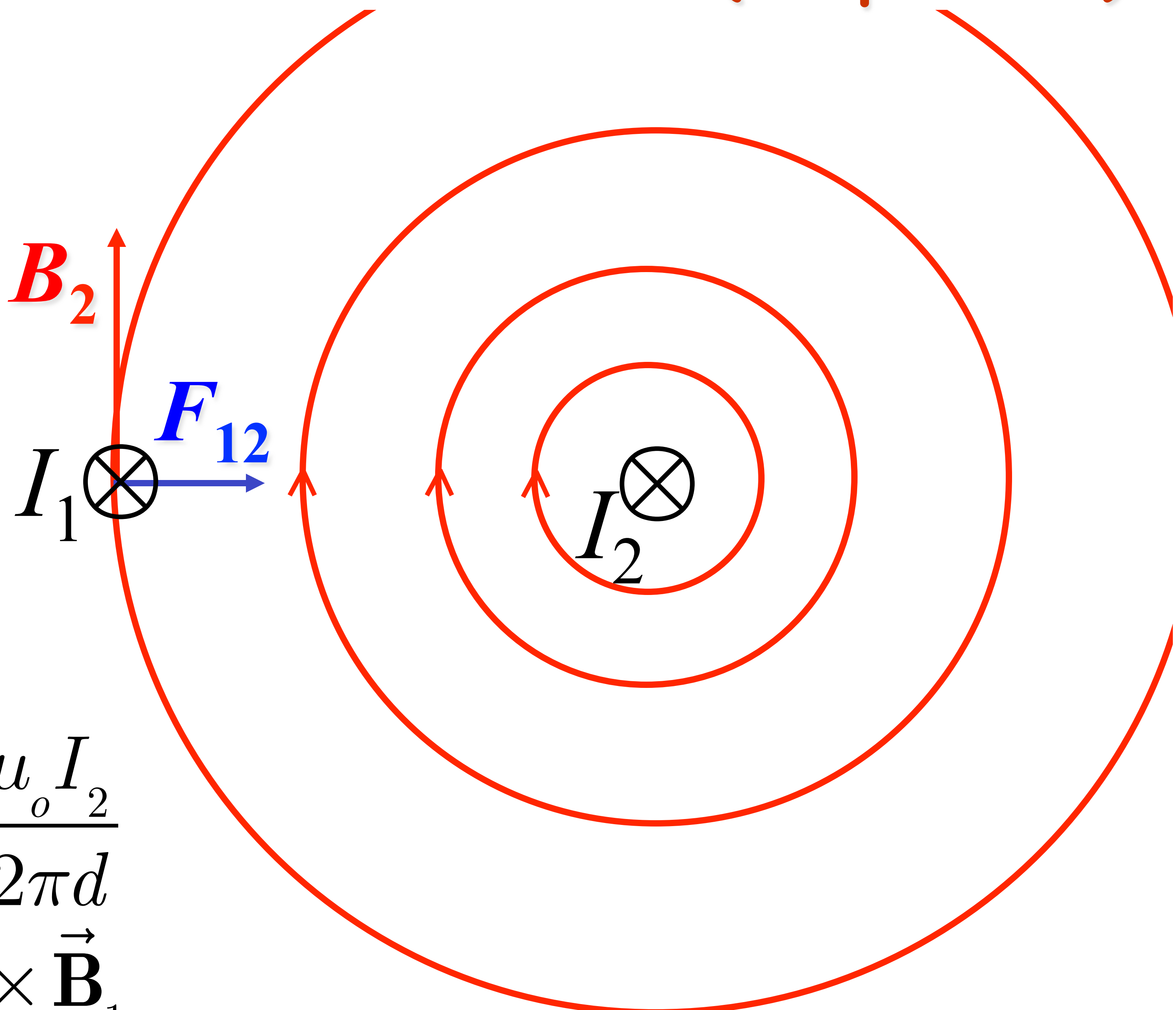
Let's look end-on at the wires (2D problem)



$$F_{21} = I_2 L \frac{\mu_o I_1}{2\pi d}$$

$$\vec{\mathbf{F}}_{21} = I_2 \vec{\mathbf{L}} \times \vec{\mathbf{B}}_1$$

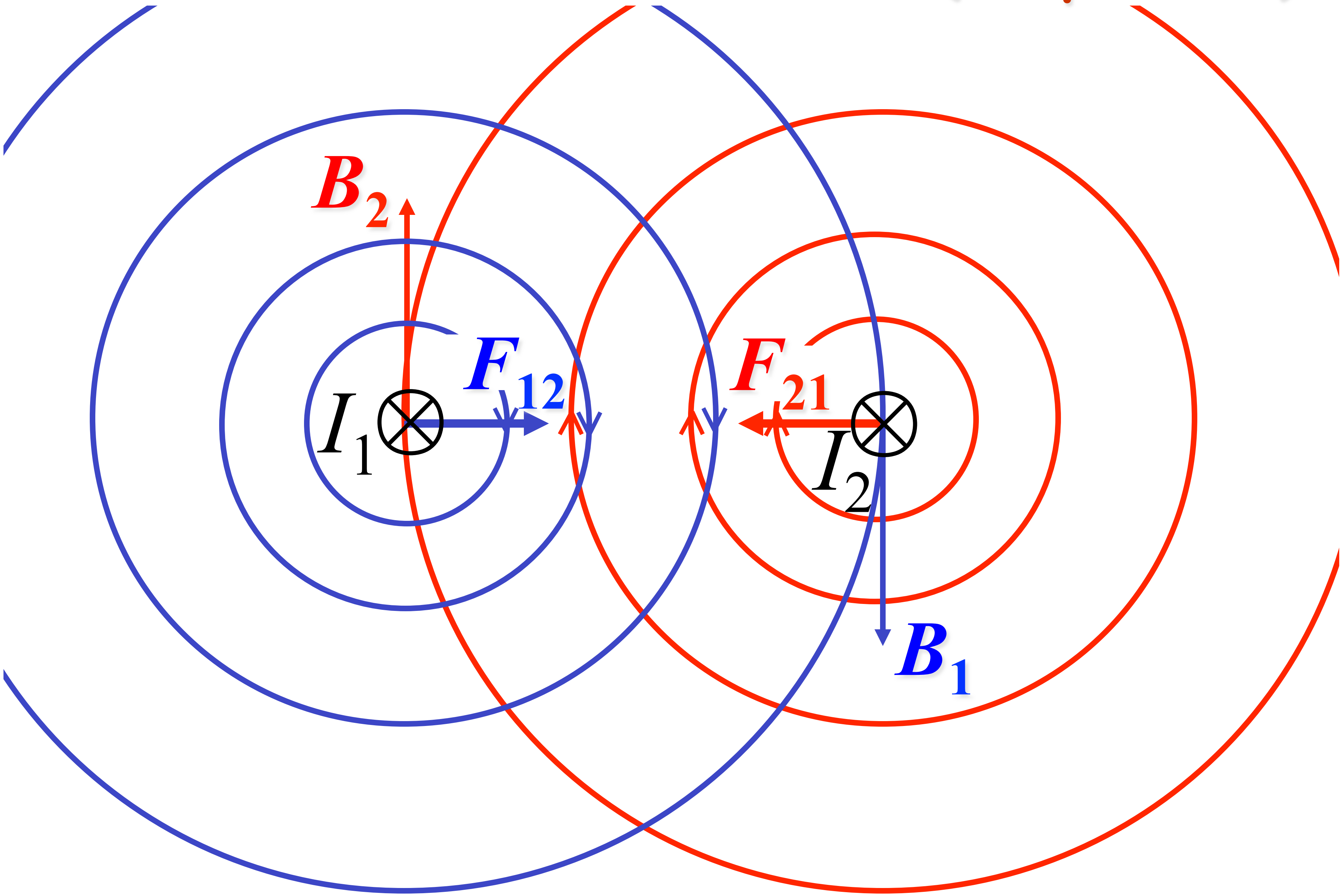
Let's look end-on at the wires (2D problem)



$$F_{12} = I_1 L \frac{\mu_o I_2}{2\pi d}$$

$$\vec{\mathbf{F}}_{12} = I_1 \vec{\mathbf{L}} \times \vec{\mathbf{B}}_1$$

Let's look end-on at the wires (2D problem)



The magnetic field due to a moving charge

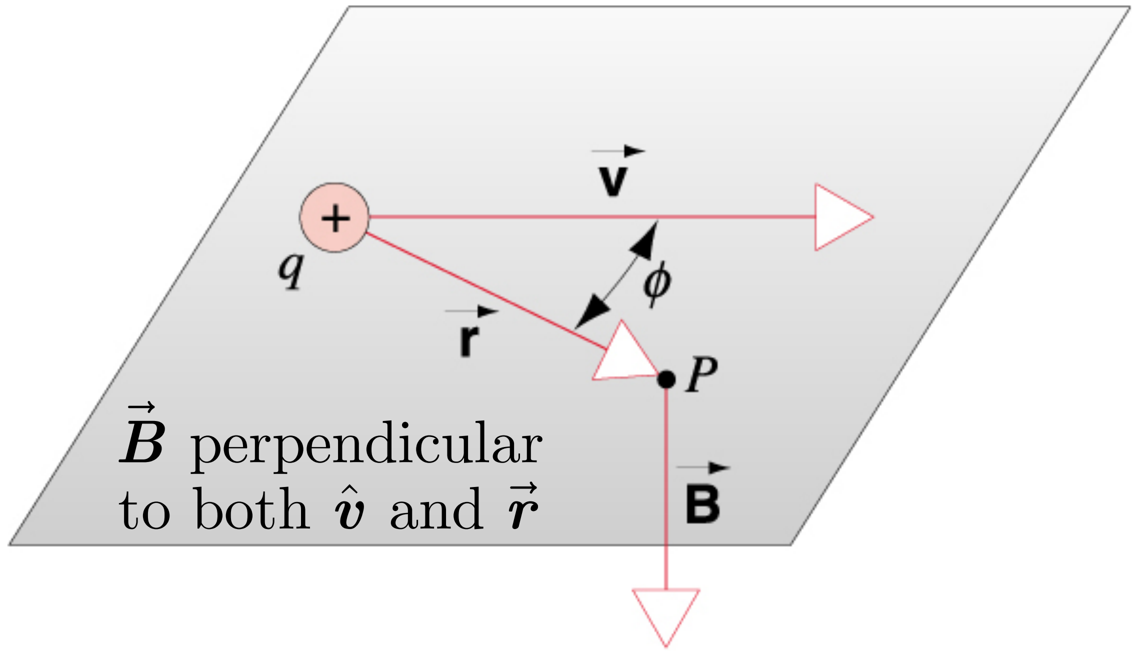
- The field strength is directly proportional to the magnitude of the velocity *v* and the charge *q*.
- If *v* reverses direction or *q* changes sign, then so does the direction of the magnetic field *B*.
- The field is zero at points along the direction of *v* (forward as well as backward).
- The field *B* is tangent to circles drawn about the velocity *v* in planes perpendicular to the velocity. The direction of *B* is determined by the right-hand-rule.
- The field decreases like $1/r^2$ along lines perpendicular to the motion of *q*.

$$\left|dB\right| = \frac{\mu_o}{4\pi}\frac{vdq}{r^2}\times\left(\text{geometrical factor}\right)$$

The magnetic field due to a moving charge

Field is maximum in plane perpendicular to velocity

$$B_{max} = \frac{\mu_o}{4\pi}\frac{vq}{r^2}$$



Like Coulomb's Law:
(For a point charge)

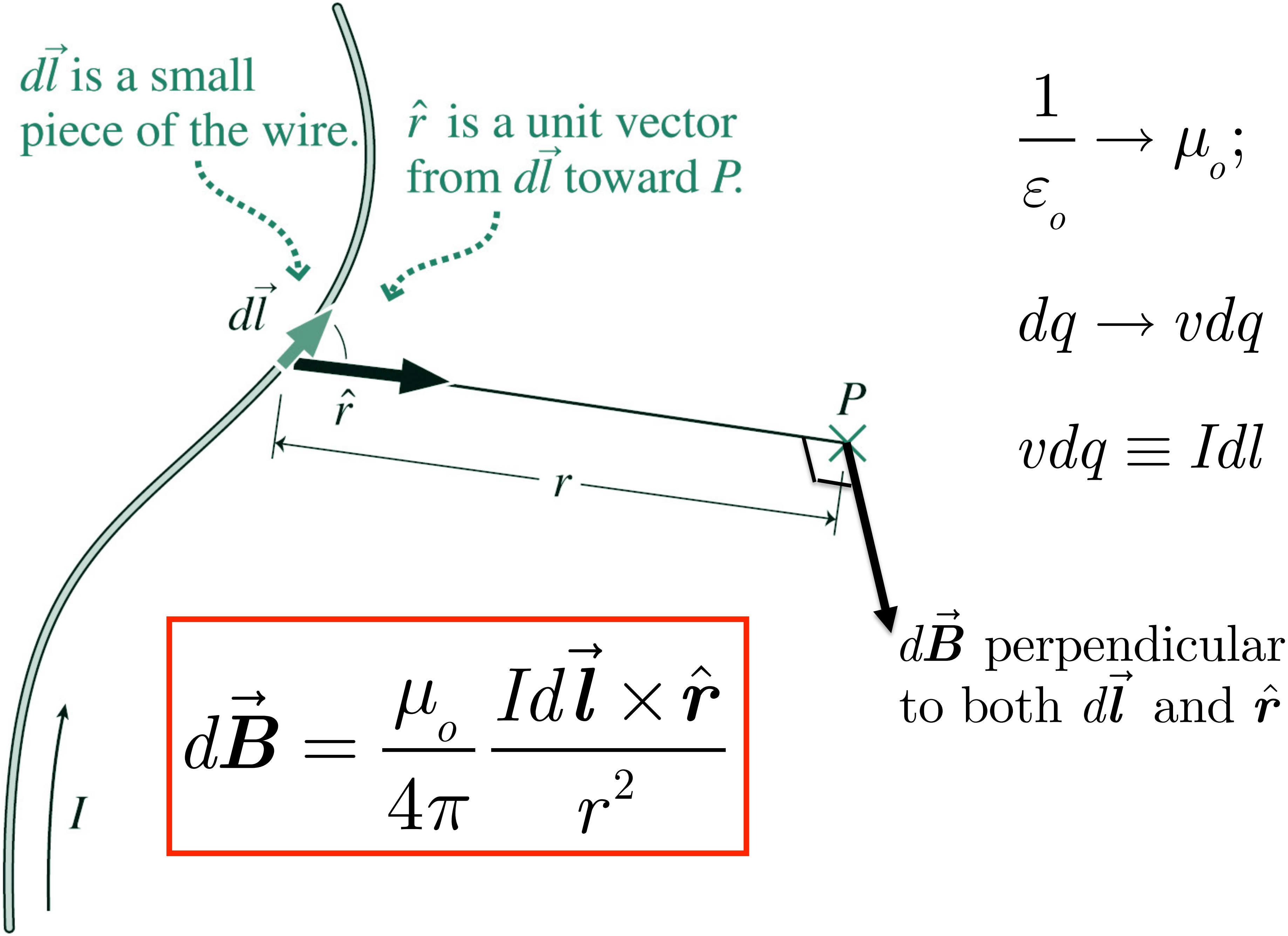
$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

$$q \rightarrow vq; \qquad \frac{1}{\varepsilon_o} \rightarrow \mu_o$$

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

$$B = \frac{\mu_o}{4\pi}\frac{|q|\,v\sin\phi}{r^2}$$

The Biot-Savart Law



$$\frac{1}{\varepsilon_o} \longrightarrow \mu_o;$$

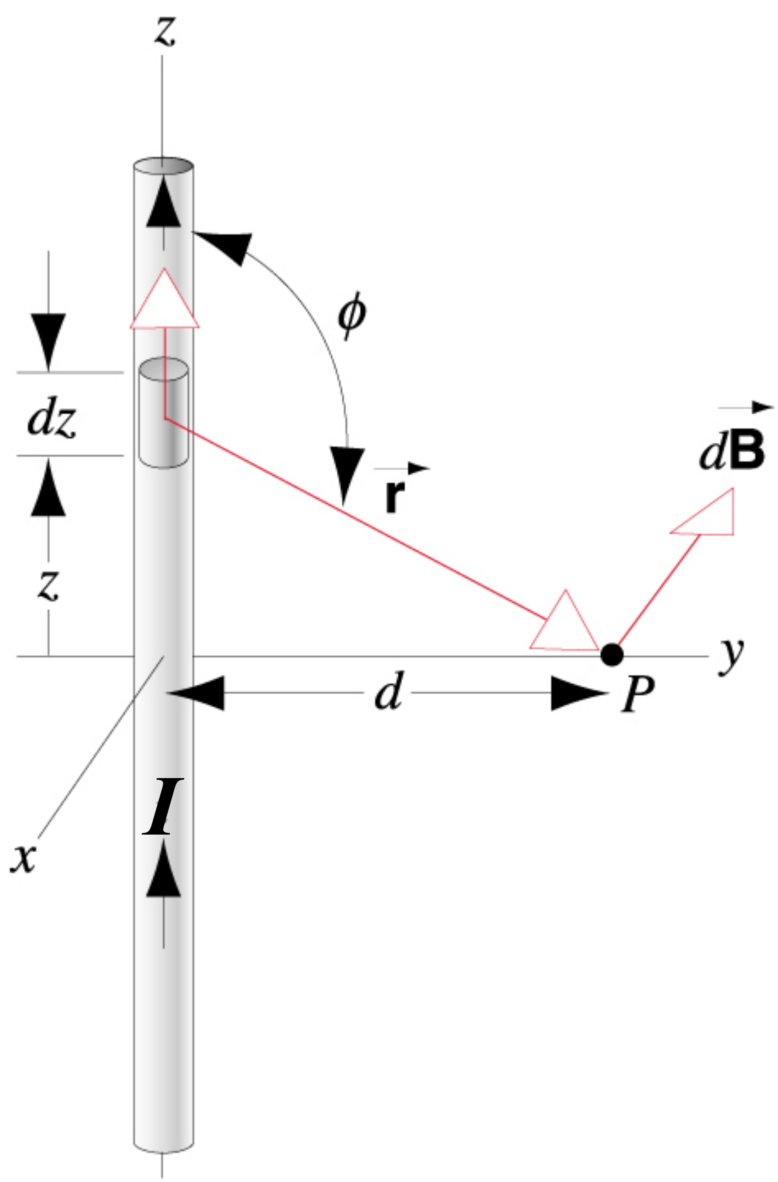
$$dq \longrightarrow v\,dq$$

$$v\,dq \equiv I\,dl$$

$$d\vec{\boldsymbol{B}} = \frac{\mu_o}{4\pi}\frac{I\,d\vec{\boldsymbol{l}} \times \hat{\boldsymbol{r}}}{r^2}$$

The Biot-Savart Law

Example 26.4



$$\vec{\mathbf{B}} = \int d\vec{\mathbf{B}}$$

$$= \frac{\mu_o}{4\pi} \int \frac{I d\vec{\boldsymbol{l}} \times \hat{\mathbf{r}}}{r^2}$$

$$= \frac{\mu_o}{4\pi} \int \frac{I dz \sin\phi}{\left(z^2 + d^2\right)} \left(-\hat{\boldsymbol{i}}\right)$$

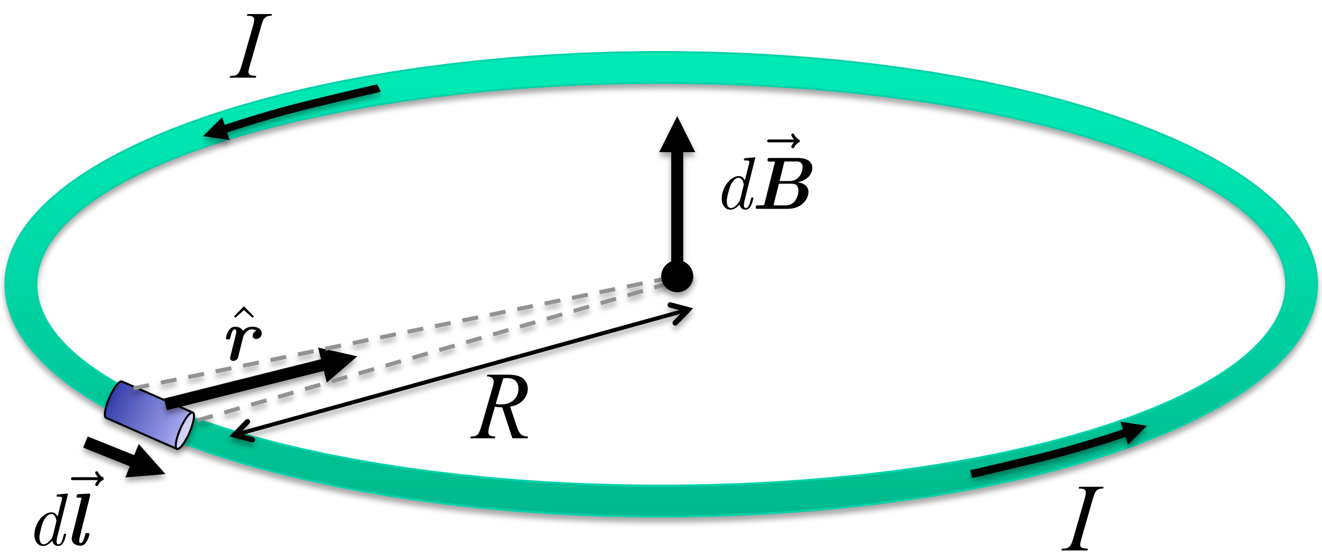
$$= \frac{\mu_o I d}{4\pi} \int_{-\infty}^{\infty} \frac{dz}{\left(z^2 + d^2\right)^{3/2}} \left(-\hat{\boldsymbol{i}}\right)$$

$$= \frac{\mu_o I}{2\pi d} \left(-\hat{\boldsymbol{i}}\right)$$

More straightforward (and important) example

Field at center of circular current loop:
Related to example 26.3

$$d\vec{\boldsymbol{B}} = \frac{\mu_o}{4\pi}\frac{Id\vec{\boldsymbol{l}} \times \hat{\boldsymbol{r}}}{r^2}$$



$$B = \frac{\mu_o I}{2R}$$

$d\vec{\boldsymbol{B}}$ perpendicular to both $d\vec{\boldsymbol{l}}$ and $\hat{\boldsymbol{r}}$